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Key Points:

- Collisional heating is the dominant process in D region electrojet modulation
- There is an azimuthal dependence to the HAARP-induced ELF/VLF source
- Vertical resonances cause amplitude enhancements near multiple of 2 kHz

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Multistation observations of the azimuth, polarization, and frequency dependence of ELF/VLF waves generated by electrojet modulation

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Abstract Modulated ionospheric heating experiments are performed with the High Frequency Active Auroral Research Program facility in Gakona, Alaska, for the purpose of generating extremely low frequency (ELF) and very low frequency (VLF) waves. Observations are made at three different azimuths from the heating facility and at distances from 37 km to 99 km. The polarization of the observed waves is analyzed in addition to amplitude as a function of modulation frequency and azimuth. Amplitude and eccentricity are observed to vary with both azimuth and distance from the heating facility. It is found that waves radiated at azimuths northwest of the facility are generated by a combination of modulated Hall and Pedersen currents, while waves observed at other azimuths are dominated by modulated Hall currents. We find no evidence for vertical currents contributing to ground observations of ELF/VLF waves. Observed amplitude peaks near multiples of 2 kHz are shown to result from vertical resonances in the Earth-ionosphere waveguide, and variations of the frequency of these resonances can be used to determine the *D* region ionosphere electron density profile in the vicinity of the HF heater.

1. Introduction

It is a significant engineering challenge to effectively radiate extremely low frequency (ELF: 3 Hz–3 kHz) and very low frequency (VLF: 3 kHz–30 kHz) electromagnetic waves. The free space wavelengths at these frequencies are on the order of tens to hundreds of kilometers, making it difficult to construct an antenna of appreciable electrical length. Even if a very long horizontal antenna is constructed, radiation in the ELF band is impeded by image currents along the Earth's surface, which is a good conductor at such low frequencies. Since both the surface of the Earth and the ionosphere are conductive, ELF/VLF radiation is effectively guided around the globe in the so-called Earth-ionosphere waveguide, which can be approximated as a parallel plate waveguide with a separation of approximately 70–85 km. The unique global propagation capability and a skin depth of at least several meters in both seawater and the ground make the ELF/VLF band of continued engineering interest despite the low data rates (or equivalently narrow bandwidths) associated with these frequencies.

Starting from the early 1970s, physicists and engineers have engaged in active experiments to modify the ionosphere with high-power high-frequency (HF: 1-10 MHz) radio waves [*Getmantsev et al.*, 1974]. Heating the ionosphere with HF waves can change the electron temperature and therefore the collision frequency within the *D* region ionosphere which is a key parameter in the plasma conductivity tensor. In the presence of natural ionospheric currents (like the auroral electrojet or equatorial electrojet), periodic modulation of the electron temperature at frequencies in the ELF/VLF band can be used to transform the ionosphere into a virtual antenna [*Stubbe et al.*, 1981, 1982; *Barr and Stubbe*, 1984a, 1984b].

A significant number of ELF/VLF generation experiments have been conducted at different ionospheric heating facilities. At the Arecibo Observatory in Puerto Rico and at the Jicamarca Observatory in Peru HF facilities were used to generate weak ELF signals with the equatorial electrojets [*Ferraro et al.*, 1982; *Lunnen et al.*, 1984]. The European Incoherent Scatter facility located in Tromsø, Norway, the High Frequency Active Auroral Research Program (HAARP), and the Sura ionospheric heating facility in Russia have been extensively utilized for the modulation of auroral electrojets [*Stubbe et al.*, 1981; *Barr and Stubbe*, 1991; *Moore et al.*, 2007;

©2015. American Geophysical Union. All Rights Reserved. *Fujimaru and Moore*, 2011; *Cohen et al.*, 2012; *Gołkowski et al.*, 2011; *Cohen and Golkowski*, 2013; *Kotik et al.*, 2013]. The two main modes of heating the ionosphere are called the ordinary mode (*O* mode) and the extraordinary mode (*X* mode). For the case of ionospheric current modulation in the *D* and lower *E* region ionosphere at subauroral latitudes where the HF carrier frequency is much greater than the local plasma frequency, these two modes are essentially left- and right-hand circularly polarized modes, respectively.

There are at least three major classes of techniques that have been investigated to modulate ionospheric currents at ELF/VLF frequencies. The first and simplest method is amplitude modulation, where the HF carrier is ON/OFF modulated (usually 50% duty cycle square wave). The unique flexibility of the HAARP ionospheric heating facility has allowed for more sophisticated modulation techniques to be explored. In the second method known as beam painting [*Papadopoulos et al.*, 1990], the HF carrier frequency is still ON/OFF modulated but the HF beam is not in a fixed stationary position. Instead, during the ON portion, the beam is quickly scanned over an area much larger than the beam width and at a rate faster than the cooling rate of the electrons. The third method is geometric modulation [*Barr et al.*, 1987; *Cohen et al.*, 2008b], where the HF beam is always ON (CW) and moved along a geometric pattern at a rate equal to the ELF/VLF modulation frequency. Amplitude modulation and continuous wave modulation techniques were compared by *Barr and Stubbe* [1997], and all three techniques are experimentally compared by *Cohen et al.* [2010b]. Additional approaches to ELF/VLF generation independent of the ionospheric currents include the so-called ionospheric current drive [*Papadopoulos et al.*, 2011] and cubic nonlinearity [*Kotik and Ermakova*, 1998; *Moore et al.*, 2013].

We focus on amplitude modulation as this is the most common technique and also most readily realized at all ionospheric modification facilities including the planned heater at the Arecibo Observatory. Several key features of ionospherically generated ELF/VLF waves have been documented. From the time of the first experiments it was found that the amplitudes of generated ELF/VLF signals exhibit maxima near multiples of 2 kHz [*Stubbe et al.*, 1982; *Papadopoulos et al.*, 2003; *Cohen et al.*, 2012]. This has been attributed to either vertical resonances of the Earth-ionosphere waveguide [*Stubbe et al.*, 1982] or as a result of optimal electron heating and cooling rates at these frequencies [*Papadopoulos et al.*, 2003]. Time of arrival analysis performed by *Fujimaru and Moore* [2011] illustrated that the typical source currents are located at altitudes of 70–80 km, consistent with earlier studies [e.g., *Rietveld et al.*, 1989]. *Cohen et al.* [2008a] demonstrated that at distances approaching 1000 km, in what can be called the far field, the current source is well approximated by a dipole oriented in the direction of the Hall current. The dependence of the generated signal strength has been shown to be a function of not only the electrojet currents but also the *D* region electron profile [*Jin et al.*, 2011; *Agrawal and Moore*, 2012].

Most previous works used observations at a single site and not at multiple sites. Consequently, the azimuthal dependence of the radiated waves has not been fully explored. Furthermore, the dominant focus of previous work has been on observed amplitude with much less analysis of the observed wave polarization, including its elliptical nature. In this context, it is also not clear which currents (Hall, Pedersen, or geomagnetically field aligned) are most responsible for observations in the near field (<150 km from the heating facility) and at what minimum distances the single dipole model of *Cohen et al.* [2008a] can be assumed to be valid [*Payne*, 2007; *Payne et al.*, 2007].

Here we present multiple site observations of the amplitude and polarization as a function of modulation frequency. The organization is as follows: section 2 describes the experiment. Observations are described and analyzed in section 3. In the fourth section we compare our observations with simulations of the ionospheric HF heating process. Discussion of the results and general conclusions are presented in the final section.

2. Experiment

The experiment was carried out with the HAARP facility located in Gakona, Alaska, during several days of the HAARP Student Summer Research Campaign in July 2011. In this experiment the HF beam was directed vertically and was modulated using square wave amplitude modulation with a carrier frequency of 2.75 MHz. The polarization of the HF heating beam alternated between X and O mode. The transmission sequence consists of 40 frequency tones chosen to avoid local power line harmonics. The exact values of the frequency tones are given in Table 1; details of the experiment times are shown in Table 2. Each tone was transmitted for 1 s. The total duration for the format (including a 20 s frequency-time ramp not discussed here) was 60 s. This pattern

Frequency 1	īones (Hz)		
500	2500	4525	6500
700	2725	4700	6700
925	2900	4900	6925
1100	3100	5125	7100
1300	3325	5300	7300
1525	3500	5500	7525
1700	3700	5725	7700
1900	3925	5900	7900
2125	4100	6100	8125
2300	4300	6325	8300

was repeated 5 times for X mode and 5 times for O mode for approximately 30 min several times between 19 July 2011 and 25 July 2011. Figure 1a shows an observed spectrogram where the tones and ramp of the format are clearly visible.

Observations were made at three sites located within 100 km of HAARP as shown in Figure 1b. The magnetic declination at the HAARP site is 21.65°. The sites are Chistochina (36 km distance, geographic azimuth: 47.7° , geomagnetic azimuth: 26.05°), Paxson (52 km distance, geographic azimuth: -20° , geomagnetic azimuth: -41.65°) and Paradise (99 km distance, geographic azimuth: 80.3°, geomagnetic azimuth: 58.65°) with respect to the HAARP facility. Signals at all sites were

recorded using ELF/VLF receivers with orthogonal air core loop antennas oriented perpendicular to the ground. The receivers at Chistochina and Paxson are of the same type as described by *Cohen et al.* [2010a].

3. Observations

3.1. Frequency Dependence of Amplitudes

The amplitudes of the received signals were analyzed as a function of the ELF modulation frequency. Transmissions using X mode yielded ELF/VLF amplitude 3-10 dB higher than O mode transmissions with all other features being approximately the same as can be seen in Figure 2, which shows the observed field magnitudes for the three sites, for X mode and O mode on 21 July 2011, 5:30 UT, normalized to the maximum observed value. Therefore, subsequent discussion here refers only to X mode transmissions. The observed calibrated magnitudes for all three sites are shown in Figure 3 as a function of frequency. Range bars indicate the standard deviation from the mean. Figure 3a shows data from 21 July 2011, 05:30 UT during which the strongest waves were observed. Figure 3b shows the observations for transmissions 3 h later, while Figure 3c shows the averaged amplitude over 6 days of transmissions. The magnitude at each site is the overall horizontal magnetic field magnitude, in other words, the square root of the sum of the squares of the amplitudes from the two antennas.

Focusing on Figure 3a, it can be seen that the highest amplitude is observed at a modulation frequency of 2.125 kHz for all three sites in line with that observed in previous work [*Stubbe et al.*, 1982; *Papadopoulos et al.*, 2003; *Cohen et al.*, 2012]. However, after the first peak observed at 2.125 kHz at all sites, the next highest peaks observed at Chistochina and Paxson are at 4.3 kHz and 6.325 kHz. The amplitudes observed at Paradise actually exhibit minima at 4.1 kHz and 6.325 kHz, and peaks shifted slightly to higher frequencies near 4.525 kHz and 7.1 kHz. We see this difference in the frequency response near 4 kHz and 6 kHz between Paradise, and the other two sites is strong evidence that the amplitude peaks are a result of the Earth-ionosphere waveguide rather than the generation process as suggested by *Papadopoulos et al.* [2003]. Furthermore, we interpret the shift of the peaks in frequency at Paradise to be indicative of a transition from locally resonant modes to propagating modes in the Earth-ionosphere waveguide. Namely, vertical resonances in the Earth-ionosphere waveguide

Table 2. Experiment Times and Solar Zenith Angles

Experiment Conditions				
Day	UT	MLT	Solar Zenith Angle	
19 July 2011	9.02-9.29	19.02–19.29	97.26°	
20 July 2011	6.30-6.59	16.30-16.59	88.64°	
21 July 2011	5.30-5.59	15.30-15.59	83.33°	
21 July 2011	8.30-8.59	18.30–18.59	96.85°	
22 July 2011	9.02-9.29	19.02-19.29	97.82°	
23 July 2011	10.10-10.40	20.10-20.40	98.56°	
24 July 2011	5.30-5.59	15.30-15.59	83.86°	





are expected to occur when the distance between the conducting ground and the conducting ionosphere are equal to a multiple of half a wavelength, while propagating modes are governed by cutoff frequencies and associated conductive losses. This identification of resonant versus propagating modes in the observations is a key conclusion of the present work, and we therefore justify our interpretation with theoretical consideration of a parallel waveguide as an approximation to the Earth-ionosphere waveguide as described below.

3.1.1. Analytical and Numerical Analysis of Resonant Versus Propagating Modes

The complex phaser amplitudes of the electric, *E*, and magnetic, *H*, fields in a parallel plate waveguide with perfectly conducting plates in the *x*-*y* plane can be written as [*Inan and Inan*, 1999, chap. 4]

$$E_{y} = E_{0} \sin\left(\frac{m\pi}{h}\right) e^{-\bar{\gamma}x} \tag{1}$$



Figure 2. Normalized amplitudes for *X* mode and *O* mode transmissions for all three sites on 21 July 2011 averaged over 5 min (5:30 UT–5:35 UT).

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$$H_{x} = \frac{jm\pi}{\omega\mu_{0}h} E_{0} \cos\left(\frac{m\pi}{h}z\right) e^{-\overline{\gamma}x}$$
(2)

$$H_z = 0 \tag{3}$$

for transverse electric (TE) modes and

$$H_{y} = \frac{\omega \epsilon h}{jm\pi} E_{0} \cos\left(\frac{m\pi}{h}\right) e^{-\bar{\gamma}x} e^{-\bar{\gamma}x}$$
(4)

$$E_{x} = E_{0} \sin\left(\frac{m\pi}{h}z\right) e^{-\overline{\gamma}x}$$
(5)

$$E_z = 0 \tag{6}$$

for transverse magnetic (TM) modes where E_0 is the arbitrary amplitude of the electric field, h is the plate separation, m is the mode number, and $\overline{\gamma} = \sqrt{\left(\frac{m\pi}{h}\right)^2 - \omega^2 \mu_0 \epsilon_0}$ is the complex propagation constant for assumed propagation in the x direction. The conditions for vertical resonance are the same as for cutoff in that $\overline{\gamma} = 0$, and there is no distinction between TE and TM modes if anisotropy of the medium is neglected. Without loss of generalization, we consider the TE mode only. The width of the resonant peaks can be obtained from the effective quality factor Q of this vertical cavity and using the universal resonance formulation [*Siebert*, 1986, p. 113]

$$|H_x(f)| \propto \frac{1}{1 + \frac{4Q^2}{f_m^2} \left(f - f_m\right)^2}$$
(7)





Figure 3. Magnitude versus frequency graphs for the three sites on (a) 21 July 2011 at 5.30 UT averaged over 5 min (5.30 UT–5.35 UT), (b) 21 July 2011 at 8.40 UT averaged over 5 mins (8.40 UT–8.45 UT), (c) averaged over 6 days (19 July 2011 to 24 July 2011). Range bars show standard deviation from the observed mean magnitude.



Figure 4. Normalized magnitude for vertical Earth-ionosphere resonances assuming reflection at altitude of 71.5 km with ionospheric conductivity of 0.09 mS/m.

where f_m is resonance frequency or cutoff frequency of mode order m. The Q of the vertical Earth-ionosphere resonator can be calculated using the average stored energy, \overline{W}_{str} , in the fields and the power loss, P_{loss} , from reflection at the Earth and ionospheric boundaries [*Inan and Inan*, 1999, chap. 5] as follows

$$Q = 2\pi f_m \frac{\overline{W}_{\text{str}}}{P_{\text{loss}}} = \pi f_m \frac{\epsilon_0 \int_V |E_y|^2 dv + \mu_0 \int_V |H_x|^2 dv}{R_s^i \int_{\text{long}} |H_y|^2 ds + R_s^E \int_{\text{Earth}} |H_y|^2 ds}$$
(8)

where $R_s^{\rm E}$ and $R_s^{\rm i}$ are the surface resistances of the Earth and ionosphere, respectively, given by $R_s = \sqrt{\pi f \mu_0/\sigma}$ where σ is the conductivity. The conductivity of the ground near Gakona, Alaska, is approximately 3 mS/m, and for this simplified analysis we use an ionospheric conductivity of 0.09 mS/m at altitude h = 71.5 km for the ionosphere, which we take to be the upper boundary of the waveguide. (As explained below, we believe that these values are the effective parameters for the observations.) Using these values, we obtain an average quality factor of 55. The associated normalized resonant curves are shown in Figure 4 and show distinct amplitude peaks at exactly 2.1 kHz, 4.2 kHz, and 6.3 kHz.

For the expected characteristics of propagating modes we consider the TE and TM mode expressions with imaginary values of $\overline{\gamma}$. We also include the effect of attenuation quantified by the attenuation constant α_c for each mode [*Inan and Inan*, 1999, pp. 276–277] given by

$$\alpha_{c_{\text{TEM}}} = \frac{R_s}{\eta h}; \quad \alpha_{c_{\text{TE}_m}} = \frac{Rs(f_m/f)^2}{\eta h \sqrt{1 - (f_m/f)^2}}; \quad \alpha_{c_{\text{TM}_m}} = \frac{2Rs}{\eta h \sqrt{1 - (f_m/f)^2}}$$
(9)

Plots of the expected horizontal magnetic field magnitude for propagating modes up to third order (m = 3) are shown in Figure 5 for three distances corresponding to the relative locations of the VLF receivers. Equal excitation of all modes has been assumed. The frequency dependence of the TE modes makes this plot of propagating waves also have peaks near multiples of 2 kHz, but these relative peaks can be seen to shift to higher frequency for farther distances since attenuation for m > 0 modes is highest when $f \simeq f_m$. Furthermore, the peaks from propagation are wider than those from resonance.

The above analytical analysis yields the expected frequency characteristic for purely resonant and purely propagating modes. To examine the more realistic case of a combination of these phenomena, we also performed a finite difference time domain (FDTD) numerical simulation. This simulation takes into account the inhomogeneity but not anisotropy of the ionosphere using a scalar conductivity derived from Profile 2 in Figure 7 as the upper boundary of a parallel plate waveguide. The waveguide was excited by orthogonal horizontal current densities of equal magnitude and phase in the *x* and *y* directions so that both TE and TM modes would be equally excited. (More accurate current distributions in the heated region are discussed



Figure 5. Normalized magnitude for propagating modes in the Earth-ionosphere parallel waveguide for different propagation distances assuming reflection at altitude of 71.5 km with ionospheric conductivity given by Profile 2 in Figure 7.

in section 4.) The source currents have a time-dependent differential Gaussian pulse with a half pulse width of 60.735 μ s giving broadband excitation over the 0.5–10 kHz band. Horizontal magnetic fields were calculated at locations corresponding to the Chistochina, Paxson, and Paradise receivers. The magnitude as a function of frequency is shown in Figure 6. It is seen that similar to the analytical results, the peak of the field magnitude shifts higher in frequency with distance from the source. For Paradise the second peak has shifted close to 4.5 kHz and the third peak close to 7.1 kHz in agreement with the observations. The width of the peaks for Paradise are also wider which is likewise in agreement with the observed frequency dependence near 4–6 kHz at Paradise.

We thus interpret the observations at Chistochina and Paxson to correspond primarily to local resonance modes, while the observations at Paradise exhibit resonance behavior near 2 kHz and mostly propagating characteristics above 3 kHz. It is reasonable to expect that the extent of a local vertical resonance in the Earth-ionosphere waveguide will be limited to the order of the size of the source and the wavelength in question. The ionospheric heated region above HAARP for a 2.75 MHz carrier has a diameter of approximately 50 km when first-order side lobes are taken into account [*Cohen*, 2009]. The free space wavelengths for 2 kHz, 4 kHz, and 6 kHz are 150 km, 75 km, and 50 km, respectively. Paradise is the most distant of the three sites (99 km versus 36 km and 52 km), so it is not surprising that the vertical resonances near 4 kHz and 6 kHz are less prominent at this site while the resonance near 2 kHz has a wavelength long enough to still be observed. Thus, for frequencies above 3 kHz the observations at Chistochina and Paxson exhibit mostly local resonance behavior for frequencies even up to about 7 kHz. For the expected vertical resonance near 8 kHz, we see little evidence of an amplitude peak at any of the sites. In the context of the expected enhancement or degradation of generated ELF/VLF waves from Earth-ionosphere waveguide effects, we see evidence that the transition



Figure 6. FDTD simulation results showing magnitude as a function of frequency for ionospheric conductivity given by Profile 2 in Figure 7. The simulation is performed with $516 \times 736 \times 468$ cells of size 0.2 km.

from enhancement at vertical resonances to degradation at the same valued mode cutoff frequencies occurs at a distance of approximately one wavelength.

3.1.2. Frequency Shift and Variability of Resonance Peaks

Close examination of the specific frequencies of the multiple amplitude peaks observed at Chistochina and Paxson shows that the second and third resonances (4.3 kHz and 6.325 kHz) are not simple multiples of the first (2.1 kHz) as would be expected for a fixed spacing between two parallel conducting planes and as was seen in Figure 4. This departure from the ideal model can be partially explained by the fact that the reflection height for 1–10 kHz waves changes slightly with frequency. Analysis of the frequency-dependent reflection height is nontrivial and was investigated extensively in the 1960s and 1970s [*Field and Engel*, 1965; *Bannister*, 1979]. We note that some more contemporary published approaches like that of *Ohya et al.* [2003], who ignore the high collision frequency in the *D* region, are not accurate in the ELF/VLF bands. Using the formulations provided by *Field and Engel* [1965] and *Bannister* [1979], the physical reflection height occurs where the conduction current is equal to the displacement current or at an altitude *z* where

$$\omega = \frac{\omega_p^2(z)}{v(z)\cos(\theta_i)} \tag{10}$$

and where v(z) is the altitude-dependent electron-neutral collision frequency, $\omega_p(z)$ is the altitude-dependent plasma frequency, ω is the wave frequency, and θ_i is the incidence angle. To investigate vertical resonances,



Figure 7. Four ionospheric *D* region profiles and electron-neutral collision frequency used to create Figure 8. Profile 1: winter night, Profile 2: summer night, Profile 3: winter day, and Profile 4: summer day [*Cohen*, 2009].



Figure 8. Expected reflection height as a function of frequency for four different ionospheric profiles and lengths of first three $\lambda/2$ multiples. Vertical resonances are expected to occur at the intersections of the curves. Observed amplitude peaks for day and night are shown as squares and circles, respectively.

we take $\theta_i = 0$. The electron density and collision frequency of four ionospheric profiles considered are presented in Figure 7. Figure 8 shows the plot of reflection height as a function of frequency for those four different ionospheric profiles. The reflection height curves in Figure 8 can be seen to have a very moderate positive slope implying that the reflection height increases slightly with frequency. Vertical resonances will occur when the reflection height is an integer multiple of one half of a wavelength, λ . The curves showing the $\lambda/2$, λ and $3\lambda/2$ lengths as a function of frequency are also plotted in Figure 8, where the reflection height and resonance curves intersect is the frequency of an expected resonance-induced amplitude enhancement. From the intersection points in Figure 8, we see that resonances at 2.125 kHz, 4.3 kHz, and 6.325 kHz are expected for ionospheric Profile 3 and correspond to the observed magnitudes from 05:30 UT on 21 July (see Figure 3a). The subauroral ionosphere over HAARP is of course variable, and this variability is also seen in the ELF/VLF observations. If we analyze the observations from a single 5 min transmission at 8:40 UT on 21 July 2011 (Figure 3b), we see observed resonances at 2.125 kHz, 4.1 kHz, and 6.1 kHz. These points are also plotted in Figure 8 and show that for this time Profile 2 is the best match to the data. The observed amplitude peaks are seen to be a function of the ionospheric profile, and the exact frequencies of peaks can potentially be used as a *D* region diagnostic.

3.1.3. Azimuthal Dependence of Observed Amplitudes

Turning to the calibrated values of the observed magnitudes in Figure 3, it is clear that the highest wave amplitudes are observed at Chistochina. Second highest magnitudes are, in general, observed at Paradise, and the lowest wave amplitudes are generally observed at Paxson. Chistochina is the closest of the three sites to HAARP; therefore, it is not surprising that the highest wave amplitudes are observed there. Paxson is located closer than Paradise to HAARP by almost 40 km, and therefore, the fact that the amplitudes generally observed at Paxson are lower than the amplitudes observed at Paradise suggests a significant azimuthal dependence of the radiation pattern. The ELF/VLF generation process involves modulation of both the Pedersen and Hall components of the ionospheric conductivity tensor. The Hall currents are typically oriented along the magnetic east-west direction (northwest-southeast geographic) with the Pedersen currents orthogonal to this. For the altitudes of interest in this paper (< 80 km) magnitude of the Hall current is typically a factor of 2 greater than the Pedersen current [*Barr and Stubbe*, 1984a, 1984b]. The analysis of polarization eccentricity and numerical simulations discussed in the following sections provides evidence that observed ELF/VLF waves at Chistochina and Paradise are sourced predominantly from the stronger Hall dipole while the waves at Paxson have components from both Hall and Pedersen currents.

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Figure 9. The normalized polarization graphs of the magnetic field from the experiment over 5 min (5.30 UT – 5.35 UT) on 21 July 2011 for the three sites, Chistochina (red), Paxson (green), and Paradise (blue).

3.2. Polarization

The most general analysis of electromagnetic polarization takes into account both the direction and the relative phasing of the field components. We follow the procedure of *Gołkowski and Inan* [2008] where the parameters of the polarization ellipse are calculated in the frequency domain from the complex signal f(t)synthesized from the two observed orthogonal components (NS(t), EW(t)) of the wave magnetic field

$$f(t) = NS(t) + iEW(t)$$
(11)

The Fourier transform $F(\omega)$ of the above complex signal f(t) is calculated, and the positive and negative frequency components can be written as follows

F

$$(\omega) = r_+ e^{i\theta_+} \tag{12}$$

and

$$F(-\omega) = r_{-}e^{i\theta_{-}} \tag{13}$$

where r_+ , r_- , and θ_+ , θ_- are the amplitudes and phases of the positive and negative ω , respectively, and also the right-hand and left-hand circularly polarized components of the signal. The geometry of the ellipse traced out by the observed wave field vectors in the time domain is defined by the lengths of the semi major axis *A* and the semi minor axis *B* and the angle ϕ measured from the geographic north-south direction. The relation between the ellipse parameters and $F(\omega)$ is as follows: ϕ is half of the difference between θ_+ and θ_- and the

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Figure 10. Eccentricity versus frequency averaged over 5 min (5.30 UT – 5.35 UT) on 21 July 2011 for the three sites, Chistochina (red), Paxson (green), and Paradise (blue).

values of *A* and *B* are defined as $A = r_+ + r_-$ and $B = |r_+ - r_-|$. The polarization ellipses obtained from the observed magnetic field for the 40 different frequency tones at the three sites are depicted in Figure 9. For each polarization ellipse we also calculate the eccentricity as

$$Eccentricity = \sqrt{1 - B^2/A^2}$$
(14)

For linear polarization the eccentricity value is 1, and for pure circular polarization the eccentricity value is 0.

Figure 10 depicts the eccentricities observed at the three sites for the 40 frequency tones transmitted. From the eccentricity curves it can be seen that waves observed at Paradise are predominantly linearly polarized as compared to Paxson and Chistochina. Chistochina exhibits linear polarization for frequencies up to 5 kHz with polarization becoming more elliptical at the higher frequencies. The ELF/VLF waves observed at Paxson are the most elliptical of the three sites. The frequencies at which the waves observed at Paxson exhibit highly linear polarization are limited to a narrow peak near 2 kHz and a broader peak between 3.8 kHz and 5.5 kHz. We believe that this is at least partly due to the vertical resonances described above that are prominent just above 2 kHz and 4 kHz. The reason that linear polarization is expected to be observed at the resonance frequencies is that reflection from the anisotropic ionosphere modifies the polarization of the generated signal. This can be seen by calculating the reflection coefficient matrix that is a function of incident angle θ_i .

$$R(\theta_i) = \begin{bmatrix} \|R_{\parallel} & \|R_{\perp} \\ \bot R_{\parallel} & \bot R_{\perp} \end{bmatrix}$$
(15)

The elements of *R* give the ratio of reflected wave amplitude to incident wave amplitude for electric field orientation parallel and perpendicular to the plane of incidence. The off diagonal terms thus quantify the conversion from parallel to perpendicular polarization. Using the method described by *Budden* [1955], the reflection matrix for vertical incidence for Profile 2 for 4.2 kHz is calculated to be

$$R(\theta_i = 0) = \begin{bmatrix} 0.8058 - 0.4843i & -0.0682 - 0.0900i\\ 0.0690 + 0.0918i & -0.9293 + 0.3193i \end{bmatrix}$$
(16)

If we start with an initially circularly polarized wave, the amplitude and polarization after ionospheric reflection is given by multiplication by *R*. Figure 11 shows the change in eccentricity that occurs after multiple reflections from the ionosphere for both a single ray and the cumulative field from the sum of all the reflected rays. Figure 11 (inset) shows the corresponding polarization ellipse. The cumulative sum of multiple rays would only be observed for vertical resonances near the source. A single ionospheric reflection changes the polarization from circular to elliptical; subsequent reflections make the polarization increase in linearity. The polarization of the cumulative wave becomes highly linear after one reflection and retains a high eccentricity (> 0.9). Therefore, at the resonance frequencies, a standing wave pattern from the summation of multiple upward and



Figure 11. Change in eccentricity of an initially right-hand circularly polarized wave upon reflections from the ionosphere described by Profile 2 in Figure 7. Dashed red trace shows eccentricity of an individual ray after undergoing multiple vertical incident reflections. Solid blue trace is the eccentricity of the cumulative wave. Inset panel shows the normalized polarization ellipse for the single ray and cumulative wave after each reflection.

downward propagating waves will yield a net linear polarization as is observed at the Paxson site. Considered in isolation, this feature of the Paxson data is further evidence of the importance of the vertical resonance phenomena in the vicinity of the heated region. At the same time, at other frequencies including near 6.5 kHz, the propagating radiation from the ionospheric source region observed at Paxson exhibits elliptical polarization can indicate the presence of two orthogonal current sources in phase quadrature. This effect is much less pronounced at the other two sites. Modulated Hall and Pedersen are spatially orthogonal, but their relative phase in time is more complicated. *Barr and Stubbe* [1984a] and *Barr and Stubbe* [1984b] found that the phase difference between modulated Hall and Pedersen currents to be either 0 or 180° depending on altitude. A phase difference of 90° would be necessary to generate circular polarization. In this context we note that the modulated currents are distributed over the heated region and additional phase differences can result from propagation. We interpret the observations as indication of radiation at Chistochina and Paradise being due to the dominant Hall dipole and the higher circularity observed at Paxson being due to contributions from both Hall and Pedersen currents.

4. Simulation of Ionospheric Current Modulation

In order to investigate the relationship between our observations and the modulated ionospheric source currents, we employ a numerical model of the ionospheric heating process. The modulation of ionospheric currents is modeled using HF ray tracing and collisional absorption providing the modified electron temperature as a function of time as has been done in other works [*Tomko et al.*, 1980; *Rodriguez and Inan*, 1994; *Cohen et al.*, 2010b; *Moore and Agrawal*, 2011]. The modified electron temperature allows for calculation of the modified Hall (σ_H) and Pedersen (σ_P) conductivities that are elements of the general plasma conductivity tensor.

$$\overline{\overline{\sigma}} = \begin{pmatrix} \sigma_{P} & -\sigma_{H} & 0\\ \sigma_{H} & \sigma_{P} & 0\\ 0 & 0 & \sigma_{||} \end{pmatrix}$$
(17)



Figure 12. (a) Amplitude versus frequency from the numerical model for selected frequencies. (b) Eccentricity versus frequency from the numerical model.

The modified conductivities lead to induced currents by way of the electrojet electric field, E,

$$\Delta \mathbf{J} = \Delta \overline{\sigma} \mathbf{E} \tag{18}$$

where the electric field is taken as 10 mV/m and assumed to be static [*Cohen et al.*, 2012] in the *y* direction corresponding to magnetic north. The model assumes a stratified ionosphere in the altitude range from 49 km to 100 km where the thickness of each layer is 1000 m. Once the currents are calculated, we derive the magnetic field radiated on the ground using the magnetic vector potential

$$\mathbf{B} = \nabla \times \mathbf{A} \tag{19}$$

The numerical model assumes a geomagnetic field in the *z* direction. A coordinate rotation is performed to take into account the magnetic zenith angle and declination at HAARP and then obtain the three components of the ELF/VLF magnetic field in geographic coordinates in the ground using

$$B_{x} = 2\frac{\mu}{4\pi}\Delta V\Sigma_{n=1}^{N} \left[J_{y_{n}}(z-z_{n}) - J_{z_{n}}(y-y_{n}) \right] \frac{1+jkR}{R^{3}} e^{-jkR}$$
$$B_{y} = 2\frac{\mu}{4\pi}\Delta V\Sigma_{n=1}^{N} \left[J_{z_{n}}(x-x_{n}) - J_{x_{n}}(z-z_{n}) \right] \frac{1+jkR}{R^{3}} e^{-jkR}$$



Figure 13. Polarization graphs from the numerical model for 12 selected frequencies.

$$B_{z} = \frac{\mu}{4\pi} \Delta V \Sigma_{n=1}^{N} \left[J_{x_{n}}(y - y_{n}) - J_{y_{n}}(x - x_{n}) \right] \frac{1 + jkR}{R^{3}} e^{-jkR} = 0$$
$$R = \sqrt{(x - x_{n})^{2} + (y - y_{n})^{2} + (z - z_{n})^{2}}$$

The presence of a conducting ground is taken into account using image theory.

We note that the numerical model takes into account only the HAARP-induced ionospheric ELF/VLF source. We do not attempt to model the propagation of the subsequent waves in the Earth-ionosphere waveguide, since our main goal is to assess the ionospheric source currents and how they determine the azimuthal dependence of the HAARP-induced ELF/VLF source. We thus do not expect to see features such as the vertical















resonance peaks in the simulated data. Figure 12 shows the simulated amplitude and eccentricity of polarization as a function of frequency for 12 frequencies. Polarization ellipses for the simulated data can be seen in Figure 13. In terms of amplitudes, Chistochina shows the highest amplitudes as was seen in the data. For frequencies above 5 kHz, the amplitudes of Paradise exceed the amplitudes of Paxson. This effect was also seen in the observations and confirms that, in general, higher amplitudes are radiated in the direction of northeastern azimuths as compared to northwestern azimuths. For polarization, the similarity between both observed and simulated results is that fields at Chistochina and Paradise show higher linearity than the fields at Paxson. Polarization observed at Paxson is more circular as compared to the other sites, suggesting that the Paxson receiver observed fields radiated by both Hall and Pedersen currents and that the relative phase and location of these currents might be conducive to circular polarization. Figure 14 shows the contour plot of the normalized simulated horizontal magnetic field strength due to Hall and Pedersen currents separately at each site for 6100 Hz. Only at Paxson is the contribution of the Pedersen current greater than that of the Hall current. The asymmetry of the patterns is due both to the magnetic declination (20.42°) and also the magnetic zenith angle (15°) as the oblique angle of the geomagnetic field affects both the modulated and image currents



Figure 16. (a) Amplitude and (b) eccentricity from the numerical model for selected frequencies for parallel conductivity ($\sigma_{||}$) modulation.

in the model. We note that an additional potential anisotropy that is neglected here is the influence of the nonvertical geomagnetic field on the HF propagation.

4.1. Effect of Modulated Geomagnetically Aligned Currents

The numerical model considers only the modulation of Hall and Pedersen currents which are close to horizontal. There have been suggestions that observed ELF/VLF generated by ionospheric modification might also result from so-called "loop back" currents, which are parallel to the geomagnetic field and arise in response the horizontal currents as a charge neutralization mechanism. In fact, it has been suggested that such currents may, in fact, dominate the observed ELF/VLF within 100 km of the HAARP facility [*Papadopoulos*, 2014]. It is important to keep in mind that the ambient parallel conductivity $\sigma_{||}$ is several orders of magnitude higher than the other elements of the conductivity tensor. An ionospheric static electric field parallel to the geomagnetic field is thus not physical, and for this reason our numerical code cannot simulate direct modulation of parallel, near vertical currents. Likewise, the model does not include strict enforcement of the continuity equation which is the basis of the loop back process [*Payne*, 2007]. Nevertheless, we can still use our model to investigate the expected effect of parallel currents on observations. A simulation was performed where only parallel currents were artificially input into the model of the same amplitude as the Hall currents in the earlier simulations. Figure 15 shows the magnitude of the observed magnetic field parallel el currents only. In Figure 16 the frequency response and polarization of the artificial parallel only simulation are shown. It can be seen that when only the parallel currents are taken into consideration, the simulation results deviate from observations significantly. For example, in the observations, the Chistochina site has the highest magnetic field amplitudes but the magnetic field generated due to the parallel currents yields amplitudes at Chistochina significantly lower than at the other two sites. Parallel currents are seen to generate a radiation null in the close vicinity of the HAARP facility, a feature that is not seen in our observations nor any other published works. The polarization results from a "parallel only" simulation are also disparate from the observations.

5. Summary and Discussion

Our key findings can be summarized as follows:

- 1. Lack of difference in ELF/VLF observations between *O* mode and *X* mode transmissions other than a simple amplitude change confirms that collisional heating is the dominant process behind *D* region electrojet modulation.
- There is an azimuthal dependence to the HAARP-induced ELF/VLF source. Waves observed in the geographic northwest azimuth have lower magnitudes and higher degree of circular polarization likely due to more equal contribution of Hall and Pedersen currents. Other azimuths have radiation dominated by Hall currents.
- 3. Amplitude enhancements of observed ELF/VLF waves near multiples of 2 kHz are due to vertical resonances between the Earth and the ionosphere. The exact frequencies of the resonances near 4 kHz and 6 kHz can be used to diagnose the *D* region ionospheric profile.
- 4. The spatial extent of the vertical resonances is on the order of a wavelength; beyond this distance observed waves show some characteristics of propagating waveguide modes.
- 5. Vertical geomagnetically aligned loop back currents play a negligible role in ELF/VLF observations on the ground for the frequencies in the explored range (500 Hz-8.3 kHz).

In the context of the third point, we note that the *D* region ionosphere is notoriously difficult to probe directly. *Gołkowski et al.* [2013] recently proposed a ELF/VLF-driven *D* region diagnostic technique using dual HF beams both AM modulated but with a ELF phase offset. While the dual beam technique is attractive since it requires observations at only a single frequency, the requirement of two simultaneous HF beams is hard to realize outside of unique facilities like HAARP. On the other hand, identification of vertical resonance frequencies requires observations at closely spaced frequencies, but the single vertical HF beam configuration is readily realizable at other facilities including the Sura heater and the planned Arecibo heating facility. Experiments at the latter in conjunction with the colocated powerful incoherent scatter radar will be the subject of future study.

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